

# Fermion Virtual Effects in $e^+e^- \rightarrow W^+W^-$ Cross Section <sup>1</sup>

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## Abstract

We analyse the contribution of new heavy virtual fermions to the  $e^+e^- \rightarrow W^+W^-$  cross section. We find that there exists a relevant interplay between trilinear and bilinear oblique corrections. The result strongly depends on the chiral or vector-like nature of the new fermions. As for the chiral case we consider sequential fermions: one obtains substantial deviation from the Standard model prediction, making the effect possibly detectable at  $\sqrt{s} = 500$  or  $1000$  GeV linear colliders. As an example for the vector-like case we take a SUSY extension with heavy charginos and neutralinos: due to cancellation, the final effect turns out to be negligible.

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We present here a study of virtual effects from new physics in the cross-section for  $e^+e^- \rightarrow W^+W^-$ . This analysis has been motivated by the following questions. What room is left for deviations with respect to the SM predictions, once the constraints coming from the LEP1 results have been properly accounted for? Are these deviations entirely due to the appearance of anomalous trilinear vector-boson couplings, as generally assumed? Is there any role played by the so-called oblique corrections?

To deal with these questions we have focused on two simple SM extensions:

- **Model 1: the SM plus heavy chiral fermions.** In particular we consider an extra doublet of heavy quarks, exact replica of the SM counterparts, as far as their electroweak and strong quantum numbers are concerned;
- **Model 2: the SM plus heavy vector-like fermions.** As an example we take the Minimal Supersymmetric Standard Model (MSSM) with heavy (above the production threshold) electroweak gauginos and higgsinos, and very heavy (decoupled) squarks, sleptons and additional higgses.

In both models the one-loop corrections to the process in question are concentrated in vector-bosons self-energies and three-point functions, which makes it possible to analyse the interplay between the two contributions [1]. Box corrections are obviously absent in the first model, while they are negligibly small in the second case, provided that the scalar masses are sufficiently large. Then the only relevant contributions remain those of gauginos and higgsinos to vertices and self-energies.

The constraints coming from LEP1 results are quite different in the two models. A mass splitting between the two new quarks, whose left-handed components are partners of a common  $SU(2)_L$  doublet, results in a potentially large contribution to the  $\epsilon_1$  parameter. To avoid disagreement with the experimental result [2], we will consider the case of degenerate quarks (a detailed discussion concerning this point is provided in [3]). A more stringent limitation comes from the  $\epsilon_3$  variable, which depends mildly on the quark masses. Each new degenerate quark doublet contributes to  $\epsilon_3$  with the positive amount

$$\delta\epsilon_3 = \frac{G_F m_W^2}{4\pi^2 \sqrt{2}} \simeq 1.3 \cdot 10^{-3}. \quad (1)$$

For  $m_t = 175$  GeV and  $m_H = 100$  GeV, a single new coloured doublet (with the possible addition of a lepton doublet) saturates the present  $2\sigma$  upper bound on  $\epsilon_3$ , and additional chiral doublets beyond the multiplet considered here are experimentally ruled out.

On the contrary, the version of the MSSM here analysed easily respects the constraints from LEP1. Indeed contributions to the  $\epsilon$  variables come in inverse power of the gaugino and higgsino masses and they turn out to be numerically quite small, within the experimental bound. For practical purposes we have considered the case of degenerate electroweak gauginos of mass  $M$  and degenerate higgsinos of mass  $\mu$ . This is the case when the mixing terms among gauginos and higgsinos are much smaller compared to the SUSY breaking gaugino mass term  $M$  and the supersymmetric  $\mu$  contribution coming from the  $H_1 H_2$  term in the superpotential.

After the inclusion of the one-loop corrections due to the new particles and of the appropriate counterterms, the reduced amplitude for the process at hand reads, following

$\lambda\bar{\lambda}$	$A_{\lambda\bar{\lambda}}^\gamma = A_{\lambda\bar{\lambda}}^Z$	$B_{\lambda\bar{\lambda}}$	$C_{\lambda\bar{\lambda}}$
$++ , --$	1	1	$1/\gamma^2$
$+0 , 0-$	$2\gamma$	$2\gamma$	$2(1+\beta)/\gamma$
$0+ , -0$	$2\gamma$	$2\gamma$	$2(1-\beta)/\gamma$
00	$2\gamma^2 + 1$	$2\gamma^2$	$2/\gamma^2$

Table 1: Standard Model coefficients expressed in terms of  $\gamma^2 = s/4m_W^2$ .

the conventions of ref. [5]:

•  $\Delta\lambda = \pm 2$

$$\tilde{\mathcal{M}} = -\frac{\sqrt{2}}{\sin^2 \bar{\theta}} \delta_{\Delta\sigma, -1} \left[ 1 - \frac{\sin^2 \bar{\theta}}{\cos 2\bar{\theta}} \Delta r_W - e_6 \right] \frac{1}{1 + \beta^2 - 2\beta \cos \Theta} \quad (2)$$

•  $\Delta\lambda = \leq 1$

$$\begin{aligned} \tilde{\mathcal{M}}^\gamma &= -\beta \delta_{|\Delta\sigma|, 1} [1 + \Delta\alpha(s)] [A_{\lambda\bar{\lambda}}^\gamma + \delta A_{\lambda\bar{\lambda}}^\gamma(s)] \\ \tilde{\mathcal{M}}^Z &= \beta \frac{s}{s - m_Z^2} \left[ \delta_{|\Delta\sigma|, 1} - \frac{\delta_{\Delta\sigma, -1}}{2 \sin^2 \bar{\theta} (1 + \Delta k(s))} \right] \cdot \\ &\quad \left[ 1 + \Delta\rho(s) + \frac{\cos 2\bar{\theta}}{\cos^2 \bar{\theta}} \Delta k(s) \right] [A_{\lambda\bar{\lambda}}^Z + \delta A_{\lambda\bar{\lambda}}^Z(s)] \\ \tilde{\mathcal{M}}^\nu &= \frac{\delta_{\Delta\sigma, -1}}{2 \sin^2 \bar{\theta} \beta} \left[ 1 - \frac{\sin^2 \bar{\theta}}{\cos 2\bar{\theta}} \Delta r_W - e_6 \right] \left[ B_{\lambda\bar{\lambda}} - \frac{1}{1 + \beta^2 - 2\beta \cos \Theta} C_{\lambda\bar{\lambda}} \right] \end{aligned} \quad (3)$$

In eq. (3)  $\beta = (1 - 4m_W^2/s)^{1/2}$ ,  $\Theta$  is the scattering angle of  $W^-$  with respect to  $e^-$  in the  $e^+e^-$  c.m. frame;  $\sigma, \bar{\sigma}, \lambda, \bar{\lambda}$  are the helicities for  $e^-, e^+, W^-$  and  $W^+$ , respectively;  $\Delta\sigma = \sigma - \bar{\sigma}$ ;  $A_{\lambda\bar{\lambda}}^\gamma, A_{\lambda\bar{\lambda}}^Z, B_{\lambda\bar{\lambda}}$  and  $C_{\lambda\bar{\lambda}}$  are tree-level SM coefficients listed in Table 1;  $\Delta\alpha(s), \Delta k(s), \Delta\rho(s), \Delta r_W$  and  $e_6$  are finite self-energy corrections [4, 7], given in the Appendix where also the effective weak angle  $\bar{\theta}$  is defined. Finally,  $\delta A_{\lambda\bar{\lambda}}^\gamma$  and  $\delta A_{\lambda\bar{\lambda}}^Z$  represent the corrections to the trilinear gauge boson vertices. For the models considered, they can be expressed in terms of the  $CP$ -invariant form factors  $\delta f_i^{\gamma, Z}$  ( $i = 1, 2, 3, 5$ ) according to the relations:

$$\begin{aligned} \delta A_{++}^V &= \delta A_{--}^V = \delta f_1^V \\ \delta A_{+0}^V &= \delta A_{0-}^V = \gamma(\delta f_3^V + \beta f_5^V) \\ \delta A_{-0}^V &= \delta A_{0+}^V = \gamma(\delta f_3^V - \beta f_5^V) \\ \delta A_{00}^V &= \gamma^2 [-(1 + \beta^2)\delta f_1^V + 4\gamma^2\beta^2\delta f_2^V + 2\delta f_3^V] \end{aligned} \quad (4)$$

Here  $\delta f_i^V$  includes both the contribution coming from the 1PI one-loop correction to the vertex  $VWW$  and the wave-function renormalization of the external  $W$  legs, taken on the mass-shell. This makes the terms  $\delta f_i^V$  finite. Other choices in the renormalization conditions are equivalent (see for example [6]).

Some comments are in order. The tree-level SM amplitudes are recovered from the above formulae by taking  $\Delta\alpha(s) = \Delta k(s) = \Delta\rho(s) = \Delta r_W = e_6 = \delta A_{\lambda\bar{\lambda}}^\gamma = \delta A_{\lambda\bar{\lambda}}^Z = 0$ . In the high-energy limit, the individual SM amplitudes from photon,  $Z$  and  $\nu$  exchange are proportional to  $\gamma^2$  when both the  $W$ 's are longitudinally polarized ( $LL$ ) and proportional to  $\gamma$  when one  $W$  is longitudinal and the other is transverse ( $TL$ ). The cancellation of the  $\gamma^2$  and  $\gamma$  terms in the overall amplitude is guaranteed by the tree-level, asymptotic relation  $A_{\lambda\bar{\lambda}}^\gamma = A_{\lambda\bar{\lambda}}^Z = B_{\lambda\bar{\lambda}}$ . When one-loop contributions are included, one has new terms proportional to  $\gamma^2$  and  $\gamma$  (see  $\delta A_{\lambda\bar{\lambda}}^\gamma$  and  $\delta A_{\lambda\bar{\lambda}}^Z$  in eq. (4)) and the cancellation of those terms in the high-energy limit entails relations among oblique and vertex corrections. We have explicitly checked that in all cases considered, this cancellation does take place. We stress that omitting, for instance, the gauge boson self-energies such cancellation does not occur any longer and the resulting amplitudes violate the requirement of perturbative unitarity.

On the other hand, one of the possibilities one can think of to have appreciable deviations in the cross-section is to delay the behaviour required by unitarity [6]. This may happen if in the energy window  $m_W \ll \sqrt{s} \leq 2M$  ( $M$  denoting the mass of the new particles) the above cancellation is less efficient and terms proportional to positive power of  $\gamma$  survive in the total amplitude. If  $\gamma$  is sufficiently large, then a sizeable deviation from the SM prediction is not unconceivable.

We now come to the quantitative discussion.

• **Model 1: Chiral fermions.**

In fig. 1 we plot  $N$ , the number of  $W^+W^-$  events per bin, versus  $\Theta$  at  $\sqrt{s} = 500$  GeV in the channel  $LL$ , taking  $M = 300$  GeV and assuming a luminosity of  $20 \text{ fb}^{-1}$ . The error bars refer to the statistical error, the full line denotes the SM expectation at tree level<sup>1</sup>, while the dashed line gives the prediction for model 1. We notice a clear indication of a significant signal, fully consistent with present experimental bounds. The departure from the SM prediction [8] becomes even more conspicuous for larger  $\sqrt{s}$ . In fig.2 we report the result for  $\sqrt{s} = 1000$  GeV,  $M = 600$  GeV and a luminosity of  $100 \text{ fb}^{-1}$ .

For understanding better this point it is useful introduce the following quantity  $\Delta R$

$$\Delta R = \frac{\left(\frac{d\sigma}{d\cos\Theta}\right) - \left(\frac{d\sigma}{d\cos\Theta}\right)_{SM}}{\left(\frac{d\sigma}{d\cos\Theta}\right)_{SM}}. \quad (5)$$

that represents the deviation between the SM and new physics differential cross-section normalized to the SM. For  $\cos\Theta$  not close to 1, and in the limit  $m_W \ll M, \sqrt{s}$  we have derived the following analytical expression for  $\Delta R_{LL}$ :

$$\Delta R_{LL} = |1 + 2K|^2 - 1 \quad (6)$$

$$K = \frac{g^2 N_c}{16\pi^2} \frac{4M^2}{s} \cdot \gamma^2 \cdot F\left(\frac{M^2}{s}\right), \quad (7)$$

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<sup>1</sup>SM corrections shift both curves in fig. 1 by the same amount. Hence, although they change the number  $N$ , they do not affect our result concerning the size of the deviation due to the presence of new physics.

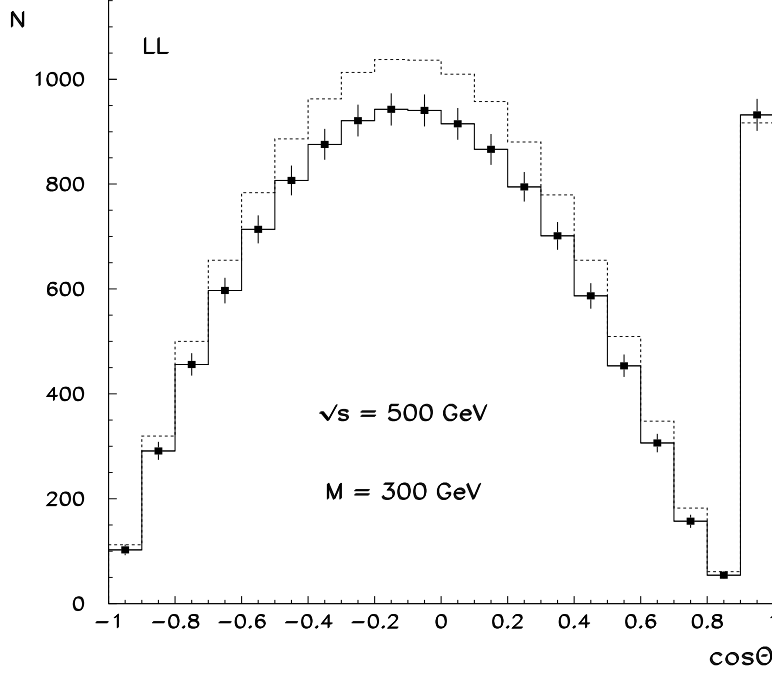


Figure 1: Number of  $W^+W^-$  events per bin versus  $\cos \Theta$  at  $\sqrt{s} = 500$  GeV in the LL channel, taking  $M = 300$  GeV and assuming a luminosity of  $20 \text{ fb}^{-1}$ .

$$F(x) = 1 - \sqrt{4x-1} \arctan \frac{1}{\sqrt{4x-1}}, \quad (8)$$

For  $s \gg M^2$ ,  $F$  grows only logarithmically and unitarity is respected. When  $M^2 \gg s$ ,  $F \simeq s/12M^2$  and the decoupling property is violated, as one expects in the case of heavy chiral fermions. In the range of energies we are interested in,  $m_W \ll \sqrt{s} \leq 2M$ ,  $K$  is of order  $G_F M^2$ , which explains the magnitude of the effect exhibited in fig. 1 and 2. A similar behaviour is also exhibited by the  $TL$  channel. Indeed, the absolute deviation in the  $TL$  channel is of  $O(\gamma)$ , but an additional  $\gamma$  factor comes from the SM cross section in the denominator of  $\Delta R$ . In fig. 3 we depict the quantity  $\Delta R$  as a function of  $\cos \Theta$  for LL channel and for the unpolarized cross section at  $\sqrt{s} = 500$  GeV. Notice that the effect which is so large in the  $LL$  channel for all the angles essentially disappears in the unpolarized cross section for  $\cos \Theta \simeq 1$  where bigger is the number of events expected. This makes it clear that for the observation of the effect under discussion good identification of the  $W$  polarization and large luminosity are essential.

• **Model 2: Vector-like fermions.**

In this case the deviations at  $\sqrt{s} = 500$  GeV or even higher energies reach at most a few per mille making it questionable to have a chance at all to observe the effect [9]. The analytic expressions for  $\Delta R$  both in the  $LL$  and in the  $TL$  channel, in the limit  $m_W \ll M, \sqrt{s}$ , are vanishing, showing that in the case of gauginos or higgsinos, which

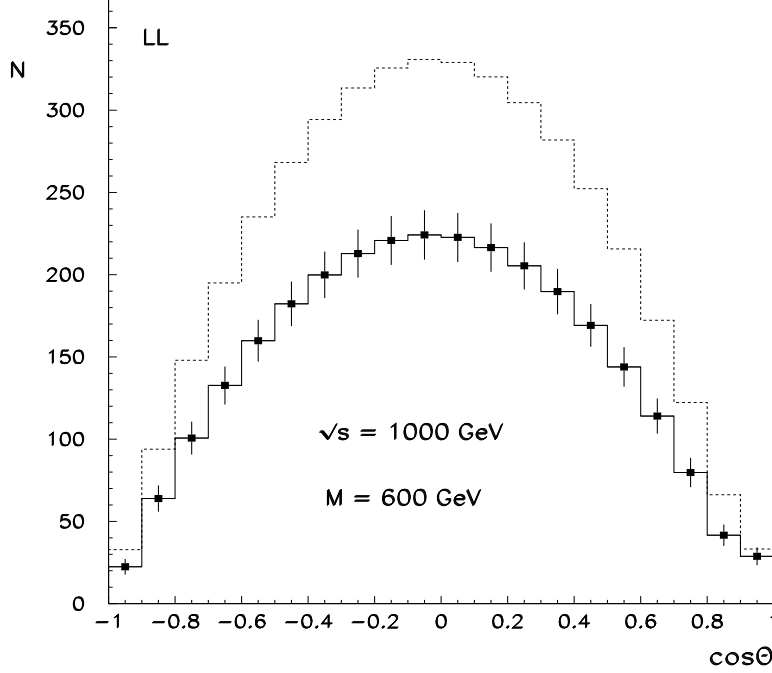


Figure 2: Number of  $W^+W^-$  events per bin versus  $\cos \Theta$  at  $\sqrt{s} = 1000$  GeV in the LL channel, taking  $M = 600$  GeV and assuming a luminosity of  $100 \text{ fb}^{-1}$ .

have vector-like coupling, no unitarity delay takes place. This is in agreement with the decoupling theorem [10].

To exemplify the danger of including in the cross-section only the contribution of trilinear anomalous gauge couplings, we have computed the  $\Delta R_{LL}$  ratio at  $\sqrt{s} = 500 \text{ GeV}$ , for model 2,  $M = 300 \text{ GeV}$  and  $\mu \gg M$ , by leaving out the contribution from the vector boson self-energies. Due to the improvident omission of an essential part of the one-loop correction, one gets an unrealistic  $\Delta R_{LL} \simeq -0.2$ , with failure of unitarity when higher energies are considered.

## Appendix

The quantities  $\Delta\alpha(s)$ ,  $\Delta k(s)$ ,  $\Delta\rho(s)$ ,  $\Delta r_W$  and  $e_6$  appearing in eqs. (2) and (3) are defined by the following relations in terms of the unrenormalized vector-boson vacuum polarizations:

$$\begin{aligned}
 e_1 &= \frac{\Pi_{ZZ}(0)}{m_Z^2} - \frac{\Pi_{WW}(0)}{m_W^2} \\
 e_2 &= \Pi'_{WW}(0) - \cos^2 \bar{\theta} \Pi'_{ZZ}(0) - 2 \cos \bar{\theta} \sin \bar{\theta} \frac{\Pi_{\gamma Z}(m_Z^2)}{m_Z^2}
 \end{aligned}$$

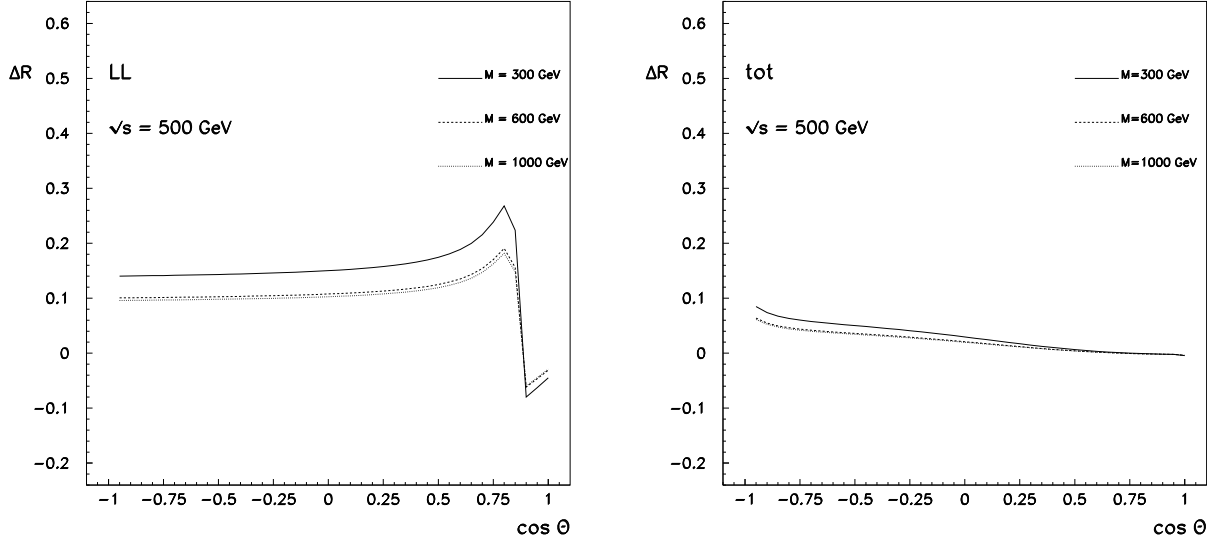


Figure 3: Relative deviation  $\Delta R$  versus  $\cos \Theta$  in the LL polarization channel and for the unpolarized cross section (tot) at  $s = (500 \text{ GeV})^2$  and  $M = 300, 600, 1000 \text{ GeV}$  for model 1.

$$\begin{aligned}
& -\sin^2 \theta \Pi'_{\gamma\gamma}(m_Z^2) \\
e_3(s) &= \frac{\cos \bar{\theta}}{\sin \bar{\theta}} \left\{ \sin \bar{\theta} \cos \bar{\theta} [\Pi'_{\gamma\gamma}(m_Z^2) - \Pi'_{ZZ}(0)] + \cos 2\bar{\theta} \frac{\Pi_{\gamma Z}(s)}{s} \right\} \\
e_4 &= \Pi'_{\gamma\gamma}(0) - \Pi'_{\gamma\gamma}(m_Z^2) \\
e_5(s) &= \Pi'_{ZZ}(s) - \Pi'_{ZZ}(0) \\
e_6 &= \Pi'_{WW}(m_W^2) - \Pi'_{WW}(0)
\end{aligned} \tag{9}$$

$$\begin{aligned}
\Delta\alpha(s) &= \Pi'_{\gamma\gamma}(0) - \Pi'_{\gamma\gamma}(s) \\
\Delta k(s) &= -\frac{\cos^2 \bar{\theta}}{\cos 2\bar{\theta}} (e_1 - e_4) + \frac{1}{\cos 2\bar{\theta}} e_3(s) \\
\Delta\rho(s) &= e_1 - e_5(s) \\
\Delta r_W &= -\frac{\cos^2 \bar{\theta}}{\sin^2 \theta} e_1 + \frac{\cos 2\bar{\theta}}{\sin^2 \theta} e_2 + 2 e_3(m_Z^2) + e_4
\end{aligned} \tag{10}$$

$$\begin{aligned}
\epsilon_1 &= e_1 - e_5(m_Z^2) \\
\epsilon_2 &= e_2 - \sin^2 \bar{\theta} e_4 - \cos^2 \bar{\theta} e_5(m_Z^2) \\
\epsilon_3 &= e_3(m_Z^2) + \cos^2 \bar{\theta} e_4 - \cos^2 \bar{\theta} e_5(m_Z^2)
\end{aligned} \tag{11}$$

with

$$\Pi'_{VV'}(s) = \frac{\Pi_{VV'}(s) - \Pi_{VV'}(m_{VV'}^2)}{(s - m_{VV'}^2)} \quad \text{with } (V, V' = \gamma, Z, W) \tag{12}$$

and  $m_{\gamma\gamma} = m_{\gamma Z} = 0$ ,  $m_{ZZ} = m_Z$  and  $m_{WW} = m_W$ . If  $s = m_Z^2$  then  $\Delta\alpha(s)$ ,  $\Delta k(s)$ ,  $\Delta\rho(s)$  coincides with the corrections  $\Delta\alpha$ ,  $\Delta k$ ,  $\Delta\rho$  which characterize the electroweak observables

at the  $Z$  resonance. Finally, the effective weak angle  $\bar{\theta}$  is defined by:

$$\sin^2 \bar{\theta} = \frac{1}{2} - \sqrt{\frac{1}{4} - \frac{\pi\alpha(s)}{\sqrt{2}G_F m_Z^2}} \quad (13)$$

where  $\alpha(s)$  is the electromagnetic coupling with all the effects coming from SM particles included at the given energy  $s$ .

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